Dipartimento di Matematica - Università di Pisa

New trends in Partial Differential Equations and Applications

organized by

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on February 20th - 21st, 2014 (Aula Magna)

Talks

Thu., Feb. 20th, 9:40–10:30 *Examples of fractional minimal surfaces*

Manuel Del Pino (Universidad de Chile)

We provide some explicit examples of nonlocal s-minimal surfaces for $0 < s < 1$, namely stationary for the s-perimeter functional, introduced first in the form of a singular integral, related to fractional Laplacian, by Caffarelli, Roquejoffre and Savin, which recovers the classical concept in the limit $s \to 1$. We find and analyze stability of a class of Lawson cones which shows important differences with the classical case. In addition we construct an axially symmetric analog of the catenoid in $\mathbb{R}^3$ in the limiting case when s approaches 1.

Thu., Feb. 20th, 10:30–11:00 *Gradient bounds and rigidity results for singular, degenerate, anisotropic partial differential equations*

Matteo Cozzi (Università degli Studi di Milano)

We consider the Wulff-type energy functional

$$\mathcal{W}_\Omega(u) := \int_\Omega B(H(\nabla u(x))) - F(u(x))\,dx,$$

where B is positive, monotone and convex, and H is positive homogeneous of degree 1. The critical points of this functional satisfy a possibly singular or degenerate, quasilinear equation in an anisotropic medium, which is

$$\operatorname{div}\left(B'(H(\nabla u))\nabla H(\nabla u)\right) + F(u) = 0.$$

We show that entire solutions of this equation satisfy a pointwise gradient estimate in the spirit of the by now classical one by Modica. Then, we use it to prove some rigidity and symmetry properties.

This is a joint work with Prof. A. Farina (Université de Picardie 'Jules Verne', Amiens) and Prof. E. Valdinoci (WIAS, Berlin).

Thu., Feb. 20th, 11:30–12:00 *Hidden convexity and applications*

Giovanni Franzina (Universität Erlangen)

We discuss several equivalent properties related to the geodesic convexity of some variational integrals and their applications to some partial differential equations. This is a joint work with Lorenzo Brasco.

Thu., Feb. 20th, 12:00–12:30 *On a long range segregation problem*

Stefania Patrizi (WIAS Berlin)

Segregation phenomena occurs in many areas of mathematics and science: from equipartition problems in geometry, to social and biological process (cells, bacteria, ants, mammals) to finance (sellers and buyers). There is a large body of literature studying segregation models where the interaction between species is punctual. There are many processes though, where the growth of a population at a point is inhibited by the populations in a full area surrounding that point.

The work we present is a first attempt to study the properties of such a segregation process. This is a joint paper with Luis Caffarelli and Veŕonica Quitalo.

Thu., Feb. 20th, 14:00–14:50 *Local and global minimality results for a nonlocal isoperimetric problem*

Massimiliano Morini (Università degli Studi di Parma)

We consider the sharp interface Ohta-Kawasaki functional related to the modeling of diblock copolymers. After reviewing the model and a local minimality criterion recently established, we present several applications to the study of the stability and the global minimality of striped patterns.

Thu., Feb. 20th, 14:50–15:20 *Symmetry and existence results for the CR-Yamabe equation*

Vittorio Martino (Alma Mater Studiorum - Bologna)

We will prove that the standard CR-Yamabe equation has infinitely many changing-sign solutions. The problem is variational but the functional fails to satisfy the Palais-Smale compactness condition; with a suitable group action we will define a subspace on which we can apply the minimax argument. The result solves a question left open from the classification of positive solutions by Jerison-Lee in the '80s.

Thu., Feb. 20th, 15:40–16:30 *Nonlocal variational problems*

Matteo Novaga (Università degli Studi di Pisa)

I will discuss a nonlocal variational model for charged liquid drops. I will also present some related nonlocal problems.

Fri., Feb. 21st, 9:40–10:30 *The CR fractional Laplacian*

María del Mar González (UPC Barcelona)

We consider the (CR) conformally covariant powers of the fractional sub-Laplacian on the Heisenberg group. As in the real case, they are given in terms of the Dirichlet-to-Neumann problem for an extension problem to the complex hyperbolic space in relation to the scattering operator. We look at some energy inequalities and applications to the CR fractional Yamabe problem. If time permits, we will consider the relation the abstract symmetric space setting.

Fri., Feb. 21st, 10:30–11:00 *A quantitative modulus of continuity in the two-phase Stefan problem*

Paolo Baroni (Uppsala Universitet)

The (two-phase) Stefan problem is a well-known mathematical model for the evolution of the configuration of a substance which is changing phase; one should think, for instance, of a piece of very cold ice melting in a lake. In this talk I am going to show how, using classic tools as the weak Harnack inequality and an appropriate alternative, one can provide an explicit, logarithmic-type modulus of continuity for local weak solutions.

This is a joint work with Tuomo Kuusi (Aalto University, Helsinki) and José Miguel Urbano (University of Coimbra).

Fri., Feb. 21st, 11:30–12:00 *Boundary regularity for fully nonlinear integro-differential equations*

Xavier Ros-Oton (UPC Barcelona)

We study fine boundary regularity properties of solutions to fully nonlinear elliptic integro-differential equations.

First, we show that the class $\mathcal{L}_0$ of Caffarelli-Silvestre is "too large" for all solutions to elliptic equations to behave comparably near the boundary. Necessary conditions for comparability of all solutions near the boundary lead to the class $\mathcal{L}_* \subset \mathcal{L}_0$,

which consists of all the infinitesimal generators of *stable* Lévy processes belonging to $\mathcal{L}_0$.

For this class of kernels $\mathcal{L}_*$ we establish fine boundary regularity results, which improve the best known ones even for linear translation invariant equations. Moreover, our estimates remain uniform as the degree of the equation approaches 2.

Fri., Feb. 21st, 12:00–12:50 *Dislocation dynamics in crystals*

Enrico Valdinoci (WIAS Berlin)

We consider an evolution equation arising in the Peierls–Nabarro model for crystal dislocation. We study the evolution of the dislocation function using analytic techniques of fractional Laplace type.

We show that, at a macroscopic scale, the dislocations have the tendency to concentrate at single points of the crystal, where the size of the slip coincides with the natural periodicity of the medium. These dislocation points evolve according to the external stress and an interior repulsive potential.

The results that will be presented have been obtained in collaboration with S. Dipierro, A. Figalli, G. Palatucci and extend previous works of R. Monneau and M. d. M. Gonzalez.